Topology Assignment 4A || Rashidul Bari

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Topology II 4A

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① what is the definition of surface?

A surface is a compact path-connected Hausdroff topological space (X, T) with the property that given any point $x \in X$, $\exists$ open set U containing x & U is homemorphic either to an open disc in $\mathbb{R}^2$ with the Euclidian topology or to an open half disc in the upper half plane with the subspace topology inherited from Eucledian topology on $\mathbb{R}^2$. A surface in space is a surface (X, T_X) where X is a subset of R^3 and T_X is subspace topology on X inherited from Euclidian topology T on R^3.

② what is the definition of surface with boundary?

let $S=(X,T)$ be a surface. A boundary point of S is a point of X for which every neighbourhood contains a half-disc like neighbourhood. The boundary of S is the set of all boundary points of S. A surface with boundary is a surface whose boundary is non-empty.

(3) Give an example of a surface and and example of a surface with boundary.

Sphere S^2 is a surface &
Cylinder is a surface with boundary.

(4) Give an example of two surfaces which are not homeomorphic.

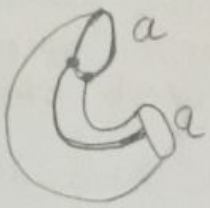
Sphere and Cylinder are Not homeomor
Since sphere is not a surface with boundary.

(5) Give an example of two surfaces with boundary which are not homeomorphic.

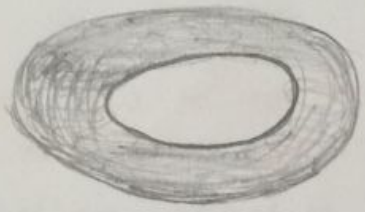
Torus with a hole on it and cylinder: cylinde
has two disjoint curves that are boundries
and mentioned torus only one.

(6) Give an example of simple & closed curve α on the torus T $\exists$ $T - \alpha$
is path connected & give an example
of simple close curve β on the
Torus T $\exists$ $T - \beta$ is not path connec

take α to be Muridian. cutting off it
we will obtain cylinder.



Take β to be a curve on the side of torus, not meridian, then cutting off it we will have tours with hole & a disk.



(8) Give an example of a surface X containing a pair of disjoint simple closed curves α & β s.t. the result of cutting X along α & β is path connected.

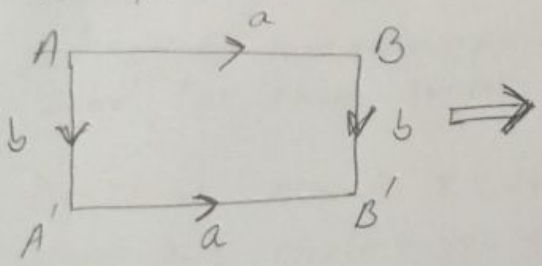
Take 2 Fold toyus & cut along 2 Meridians — one on the first hole other on the another

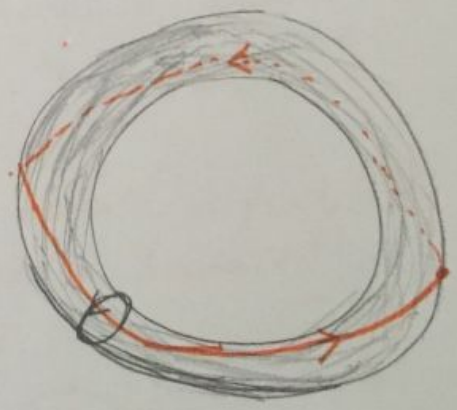
(9) what surface do you get when you glue together the boundaries of two disks?

Each disk is homeomorphic to a hemisphe (by the projection of hemisphere on the underline disk). Let one disk be an upper hemisphere and other one - a lowe Then glue together it will be a whole sphere.

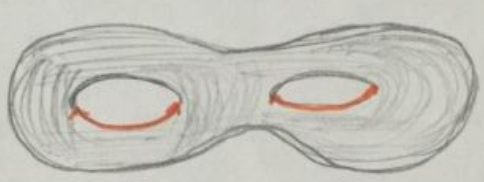
(10) Describe how to construct a torus T as the quotient space of saware(rectangl

Glue together opposite sides of the rectangl then glue together opposite boundaries of cylinder:

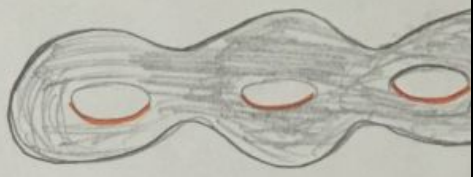




(11) Sketch Pictures of the surfaces.
$T \# T$ and $T \# T \# T$



2-Fold torus



3 Fold torus

(12) What surface results from taking the connected sum $S^2 \# S^2$ of two spheres. What about the connected sum $S^2 \# S^2 \# S^2$ of three spheres?

Connected sum of 2 spheres is again a surface homeomorphic to sphere. The same for three sphere.

(13) Describe how to "cut open" a torus T along two circles to get a square?